7.1: Absolute Value

For a real number *a*, the **absolute value** is always the non-negative value of the number. We show absolute value with two vertical lines, like brackets.

Ex. 1: $|7| =$ $|-7| =$ $-|7| =$ $|0| =$

Ex. 2: Write the following real numbers in order from least to greatest.

$|-6.5|,\ 5,\ |4.75|,\ -3.4,\ \left|-\frac{12}{5}\right|,\ |-0.1|$

We treat absolute value symbols just like brackets. Use the order of operations.

Ex. 3: Evaluate $4-|3(2)-1|+3$

Your Turn

Evaluate the following:

(a) $|4| - |-6|$

(b) $5 - 3|2 - 7|$

(c) $|-2(5-7)^2 + 6|$

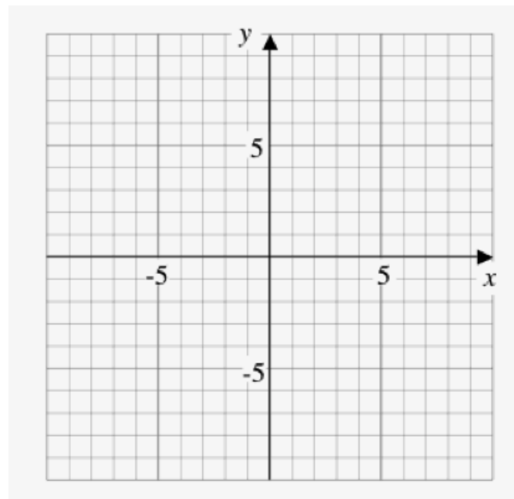
When could absolute values be useful in real life?

7.2: Absolute Value Functions

Ex. 1: Graph the functions $y = x$ and $y = |x|$ using a table of values. State both the domain and range of both graphs.

x	y

x	y



Domain: Domain:

Range: Range:

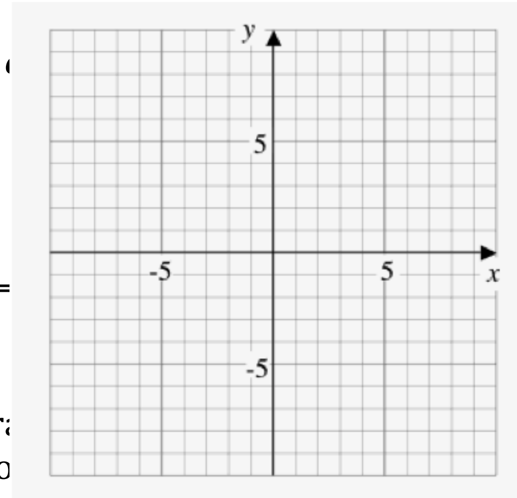
Absolute values will require the use of **piecewise notation**. This is because the function is made up of two or more separate functions with its own domain and range. They will combine to the overall functions.

- What is the piecewise notation for the above graph $y = |x|$?

In general: $|a| = \begin{cases} a, a \geq 0 \\ -a, a < 0 \end{cases}$

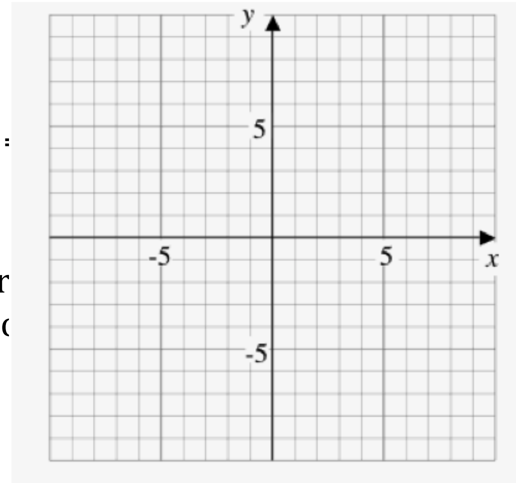
Ex. 2: Consider the absolute value function $y =$

(a) Determine the x and y intercepts.
(b) Sketch the graph.
(c) State the domain and range of the gra
(d) Express the graph with piecewise no



An **<u>invariant point</u>** is any point that remains unchanged when a transformation is applied to it.

- Can you name some invariant points in the above example?



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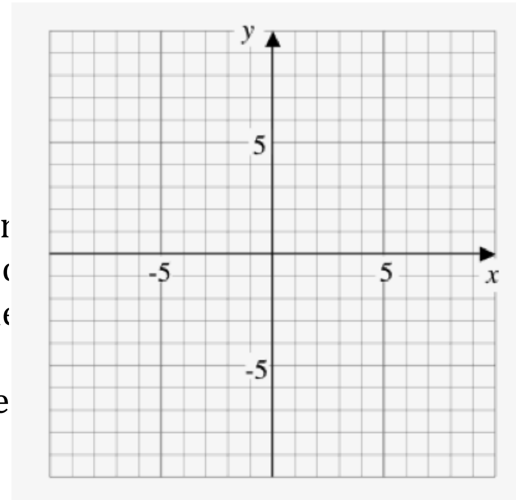
7.3: Absolute Value Equations (Part I)



When solving an absolute value equation we r ies:

1. The value inside the absolute value is pc
2. The value inside the absolute value is ne

Consider $|x| = 10$. What is a possible

Ex. 1: Solve $|x - 3| = 7$ both algebraically and graphically.

Solving absolute value equations:

1. Consider the positive and negative case for each absolute value:
   +CASE: remove absolute value bars
   - CASE: multiply the contents of the absolute value bars by -1
2. Solve each case.
3. Check solution(s) by substituting the solution back into the ORIGINAL equation. Reject any that do not work (**extraneous roots!**).

Ex. 2: Solve $|2x-5| = 5-3x$

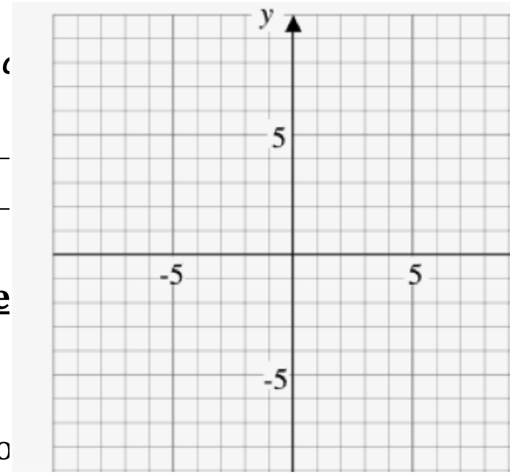
Solve $|x+5| = 4x-1$

Ex. 3: Solve $|3x-4|+12=9$

Ex. 4: Solve $|x-10|=x^2-10x$

Ex. 5: Solve $|x^2 - 2x| = 1$

Ex. 6. Solve $6|x\text{-}2|\text{-}15 = -9|x\text{-}2| + 15$



7.4: Reciprocal Functions (Part I)

Given a function $y = f(x)$, its corresponding **re**

Ex.1: Determine the reciprocal of each functio lues?

(a) $y = 2x + 3$

(b) $y = \dfrac{3}{x-1}$

Ex. 2: Sketch the graph of $y = x$ and its reciprocal function using a table of values.

$y = x$

x	y

$y = \dfrac{1}{x}$

x	y

A few questions about example 2:

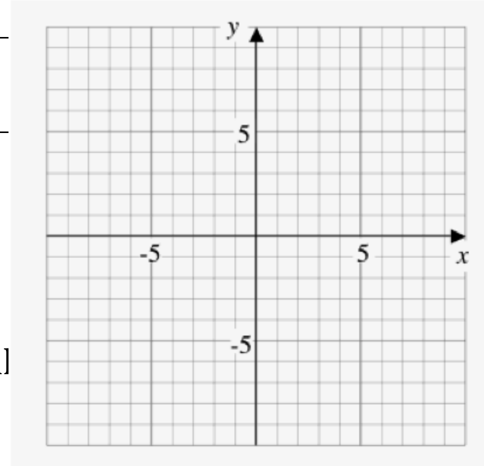
- Why does the curve approach the y-axis, but never touch it?
- Why does the curve approach the x-axis, but never touch it?
- Recall that **invariant points** are those that are unchanged. What are the invariant points for this pair of functions? What is special about the reciprocals of these values?

Asymptote: A line whose distance from a curve approaches zero.

Ex. 3: Complete the following table:

Characteristic	$y = x$	$y = \frac{1}{x}$
Domain		
Range		
End Behaviour (Quadrants)		
Behaviour at x = 0		

<u>7.4: Reciprocal Functions (Part II)</u>



Ex.1: Consider $y = x - 3$

(a) Determine its reciprocal function, $y =$

(b) Determine the equation of the vertical

(c) Graph $y = \frac{1}{f(x)}$ - start with $y = f(x)$!

* Check on your graphing calculator

<u>Graphing reciprocal functions:</u>

1. Graph the function $f(x)$. Mark the x-intercept(s) and points where $f(x) = \pm 1$ (**invariant points**).
2. Mark the **vertical asymptotes** of the reciprocal function at the x-intercepts.
3. Create the graph through the invariant points by **curving <u>with</u> the vertical asymptotes and the x-axis, not against**. (Check with a table of values or graphing calculator if needed.)

Ex.2: Consider $f(x) = x^2 - 4$

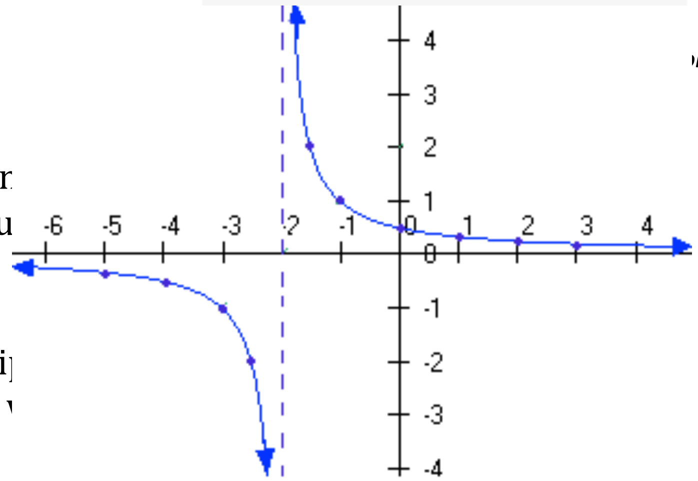
(a) What is the reciprocal functior

(b) State the non-permissible valu

asymptote(s) of $y = \frac{1}{f(x)}$.

(c) Graph the function and its reci

(d) What are the invariant points



Ex.3: Given the graph of a reciprocal function of the form $y = \frac{1}{mx + b}$:

(a) Sketch the graph of the original function $y = f(x)$.

(b) Determine the original function $y = f(x)$.